Application of Physics II for

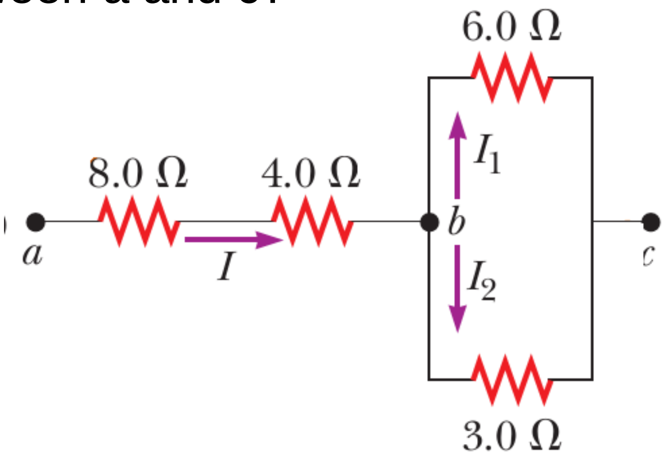
Final Exam

Question 1

Four resistors are connected as shown in Figure.

(A)Find the equivalent resistance between points a and c.

(B)What is the current in each resistor if a potential difference of 42 V is maintained between a and c?

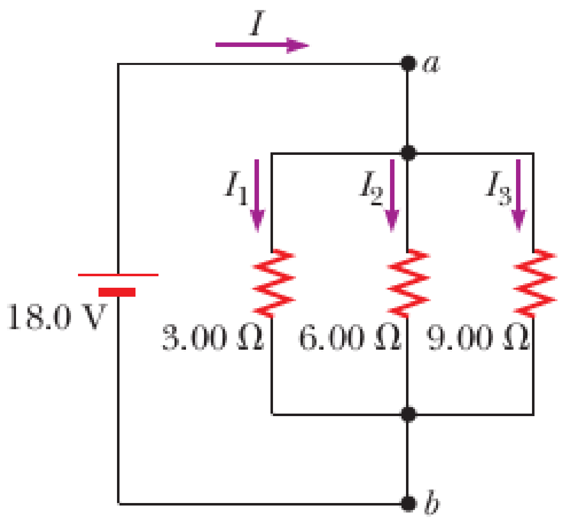


Question 2

Three resistors are connected in parallel as shown in Figure. A potential difference of 18.0 V is maintained between points a and b.

(A) Calculate the equivalent resistance of the circuit.

(B) Find the current in each resistor.

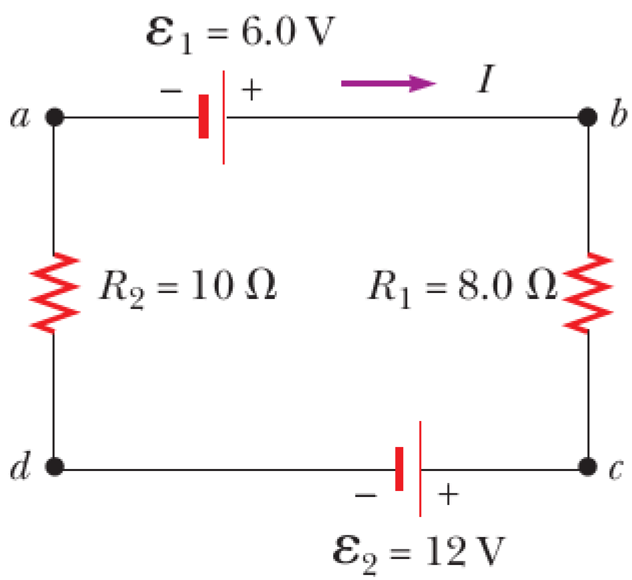


Question 3

A single-loop circuit contains two resistors and two batteries as shown in Figure. (Neglect the internal resistances of the batteries.)

a) Find the current in the circuit.

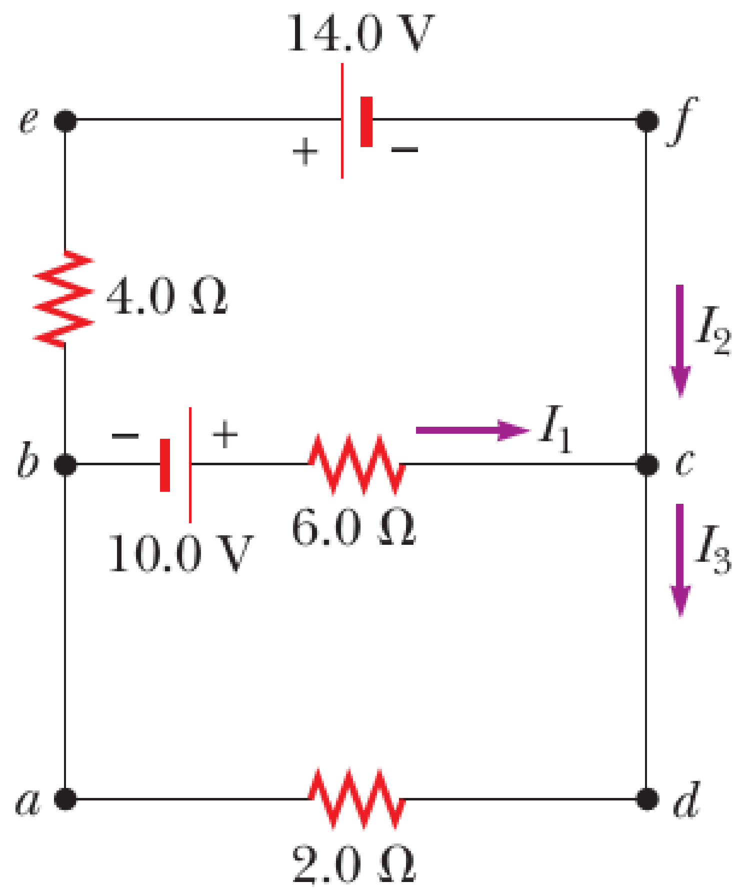
b) What is the power, spended in each resistors.



Question 4

Find the currents I1, I2, and I3

in the circuit shown in Figure.



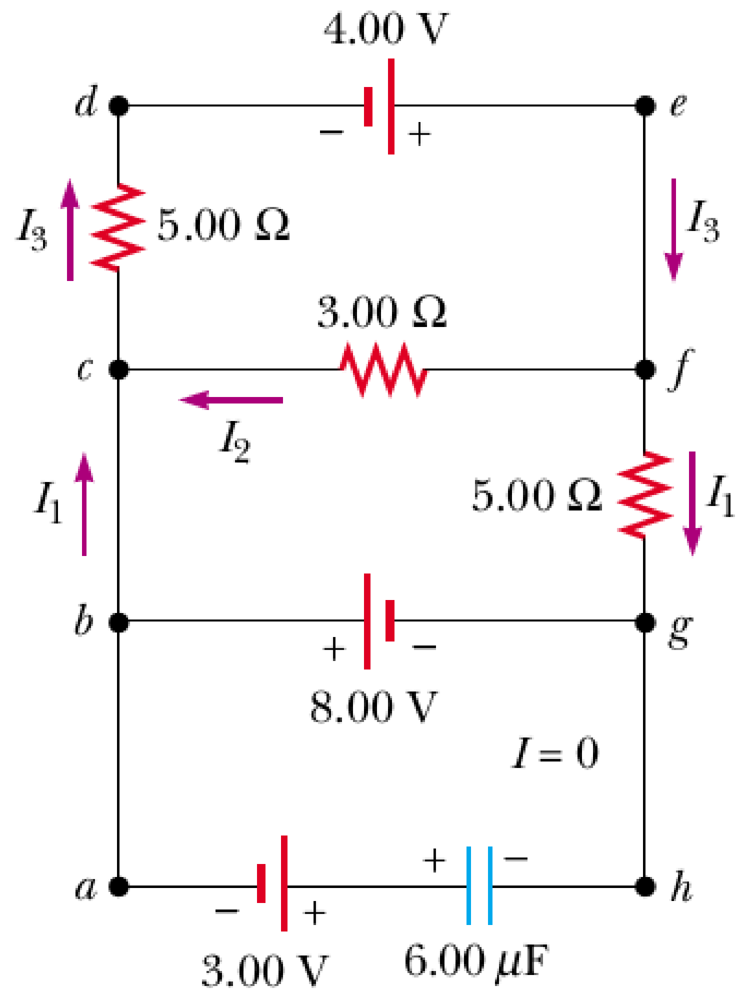
Question 5

Under steady-state conditions, find the

unknown currents I1, I2, and I3 in

the multiloop circuit shown in Figure.

What is the charge on the capacitor?



SECOND WEEK

Question 1

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

(A) Find its capacitance. (k=3.7 for paper)

(B) What is the maximum charge that can be placed on the capacitor? (dielectric strength of paper 16×10^6 V/m.)

Solution 1

$$C = \kappa \, \frac{\epsilon_0 A}{d}$$

$$= 3.7\left(\frac{(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})\,(6.0 \times 10^{-4}\ \mathrm{m^2})}{1.0 \times 10^{-3}\mathrm{m}}\right)$$

$$= 20 \times 10^{-12}\,\mathrm{F} = \boxed{20\ \mathrm{pF}}$$

Solution 1

$$\Delta V_{\max} = E_{\max} d = (16 \times 10^{6}\ \mathrm{V/m})(1.0 \times 10^{-3}\ \mathrm{m})$$
$$= 16 \times 10^{3}\ \mathrm{V}$$

Hence, the maximum charge is

$$Q_{\max} = C\,\Delta V_{\max} = (20 \times 10^{-12}\ \mathrm{F})(16 \times 10^{3}\ \mathrm{V})$$
$$= \boxed{0.32\ \mu\mathrm{C}}$$

Question 2

An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90×10^4 N/C at a distance 21.0 cm from its center.

(a) What is its surface charge density?

(b) What is its capacitance?

Solution 2

$$E = \frac{k_e q}{r^2}: \qquad q = \frac{\left(4.90 \times 10^4 \text{ N/C}\right)(0.210 \text{ m})^2}{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)} = 0.240\ \mu\text{C}$$

(a) $$\sigma = \frac{q}{A} = \frac{0.240 \times 10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C}/\text{m}^2}$$

(b) $$C = 4\pi \in_0 r = 4\pi\left(8.85 \times 10^{-12}\right)(0.120) = \boxed{13.3 \text{ pF}}$$

Question 3-4

Two capacitors, $C1=5.00\ \mu F$ and $C2=12.0\ \mu F$, are connected in parallel, and the resulting combination is connected to a 9.00-V battery.

(a) What is the equivalent capacitance of the combination?

(b) What are the potential difference across each capacitor and (c) the charge stored on each capacitor?

What If ? The two capacitors of above the Problem are now connected in series and to a 9.00-V battery.

Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

Solution 3

(a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}.$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\ \text{V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$

and $Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

Solution 4

(a) In series capacitors add as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$$

and $C_{eq} = \boxed{3.53\ \mu\text{F}}$.

(c) The charge on the equivalent capacitor is $Q_{eq} = C_{eq}\Delta V = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$.

Each of the series capacitors has this same charge on it.

So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$.

(b) The potential difference across each is $\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}}$

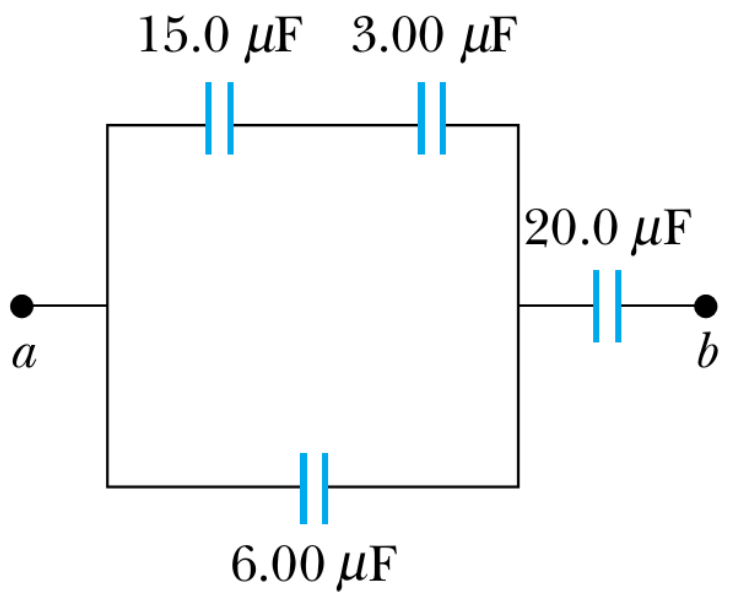
and $\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$.

Question 5

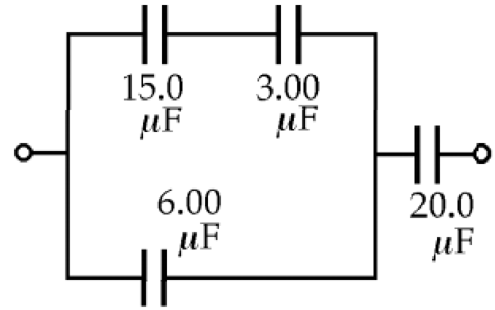
Four capacitors are connected as shown in Figure.

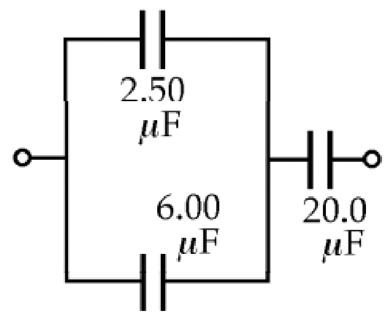
(a) Find the equivalent capacitance between points a and b.

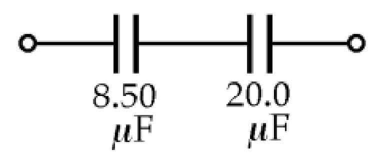
(b) Calculate the charge on each capacitor if ΔV_{ab}=15.0 V.



Solution 5







(a) $$\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$$

$$C_s = 2.50\ \mu\text{F}$$

$$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$$

$$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$$

(b) $$Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}\ \text{on}\ 20.0\ \mu\text{F}$$

$$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$$

$$15.0 - 4.47 = 10.53\ \text{V}$$

$$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}}\ \text{on}\ 6.00\ \mu\text{F}$$

$$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}\ \text{on}\ 15.0\ \mu\text{F}\ \text{and}\ 3.00\ \mu\text{F}$$

Question 6

An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate

(a) the electric field between the plates,

(b) the surface charge density,

(c) the capacitance, and

(d) the charge on each plate.

Solution 6

(a) $\Delta V = Ed$

$$E = \frac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$$

(b) $E = \dfrac{\sigma}{\epsilon_0}$

$$\sigma = \left(1.11 \times 10^{4}\ \text{N/C}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right) = \boxed{98.3\ \text{nC/m}^2}$$

(c) $$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)\left(7.60\ \text{cm}^2\right)\left(1.00\ \text{m}/100\ \text{cm}\right)^2}{1.80 \times 10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$$

(d) $\Delta V = \dfrac{Q}{C}$

$$Q = (20.0\ \text{V})\left(3.74 \times 10^{-12}\ \text{F}\right) = \boxed{74.7\ \text{pC}}$$

Question 7

In a particular cathode ray tube, the measured beam current is 30.0 A.

How many electrons strike the tube screen every 40.0 s?

Solution 7

$$I = \frac{\Delta Q}{\Delta t} \qquad \Delta Q = I\Delta t = \left(30.0 \times 10^{-6}\ \text{A}\right)(40.0\ \text{s}) = 1.20 \times 10^{-3}\ \text{C}$$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = \boxed{7.50 \times 10^{15}\ \text{electrons}}$$

Question 8

Calculate the current density in a gold wire at 20 C, if an electric field of 0.740 V/m exists in the wire.

Solution 8

$$J = \sigma E = \frac{E}{\rho} = \frac{0.740\ \text{V/m}}{2.44 \times 10^{-8}\ \Omega \cdot \text{m}} \left(\frac{1\ \Omega \cdot \text{A}}{1\ \text{V}} \right) = \boxed{3.03 \times 10^{7}\ \text{A/m}^2}$$

Question 9

A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2 . What is the current in the wire?

Solution 9

$$\Delta V = IR$$

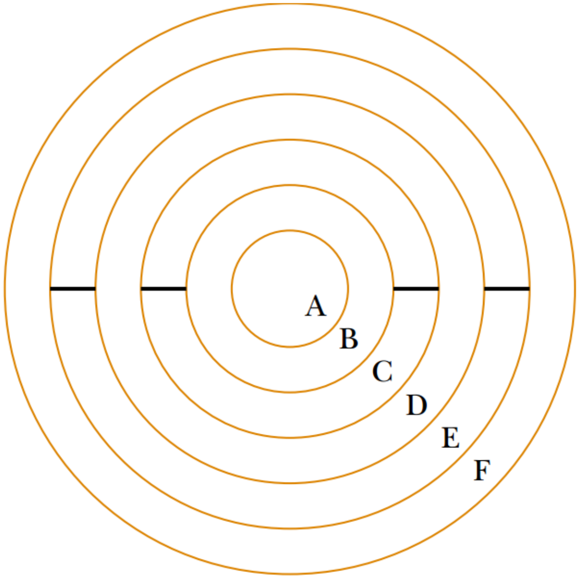
and $\qquad R = \frac{\rho \ell}{A}: \qquad A = (0.600\ \text{mm})^2 \left(\frac{1.00\ \text{m}}{1\,000\ \text{mm}} \right)^2 = 6.00 \times 10^{-7}\ \text{m}^2$

$$\Delta V = \frac{I\rho\ell}{A}: \qquad I = \frac{\Delta V A}{\rho \ell} = \frac{(0.900\ \text{V})\left(6.00 \times 10^{-7}\ \text{m}^2\right)}{\left(5.60 \times 10^{-8}\ \Omega\cdot\text{m}\right)(1.50\ \text{m})}$$

$$I = \boxed{6.43\ \text{A}}$$

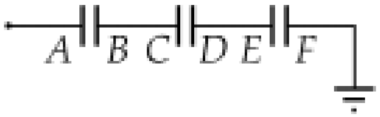
Question 10

Figure shows six concentric conducting spheres, A, B, C, D, E, and F having radii R, $2R$, $3R$, $4R$, $5R$, and $6R$, respectively. Spheres B and C are connected by a conducting wire, as are spheres D and E. Determine the equivalent capacitance of this system.



Solution 10

Positive charge on A will induce equal negative charges on B, D, and F, and equal positive charges on C and E. The nesting spheres form three capacitors in series. From Example 26.3,



$$C_{AB} = \frac{ab}{k_e(b-a)} = \frac{R(2R)}{k_e R} = \frac{2R}{k_e}$$

$$C_{CD} = \frac{(3R)(4R)}{k_e R} = \frac{12R}{k_e}$$

$$C_{EF} = \frac{(5R)(6R)}{k_e R} = \frac{30R}{k_e}$$

$$C_{\text{eq}} = \frac{1}{k_e/2R + k_e/12R + k_e/30R} = \boxed{\frac{60\,R}{37\,k_e}}$$

Question 11

Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line. Assume the cost of energy from the power company is \$0.0600/kWh

Solution 11

$$\mathcal{P} = I(\Delta V) = (1.70\ \text{A})(110\ \text{V}) = 187\ \text{W}$$

$$\text{Energy used in a 24-hour day} = (0.187\ \text{kW})(24.0\ \text{h}) = 4.49\ \text{kWh}$$

$$\therefore \text{cost} = 4.49\ \text{kWh}\left(\frac{\$0.060\,0}{\text{kWh}}\right) = \$0.269 = \boxed{26.9¢}$$

Question 12

The heating element of a coffee maker operates at 120 V and carries a current of 2.00A. Assuming that the water absorbs all of the energy delivered to the resistor, calculate how long it takes to raise the temperature of 0.500kg of water from room temperature (23.0° C) to the boiling point.

Solution 12

$$\mathcal{P} = I\Delta V = (2.00\text{ A})(120\text{ V}) = 240\text{ W}$$

$$\Delta E_{\text{int}} = (0.500\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(77.0^\circ\text{C}) = 161\text{ kJ}$$

$$\Delta t = \frac{\Delta E_{\text{int}}}{\mathcal{P}} = \frac{1.61\times 10^5\text{ J}}{240\text{ W}} = \boxed{672\text{ s}}$$

Question 13

The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds.

(a) What is the instantaneous current through the surface at t =1.00s?

(b) What is the value of the current density?

Solution 13

$$q = 4t^3 + 5t + 6$$

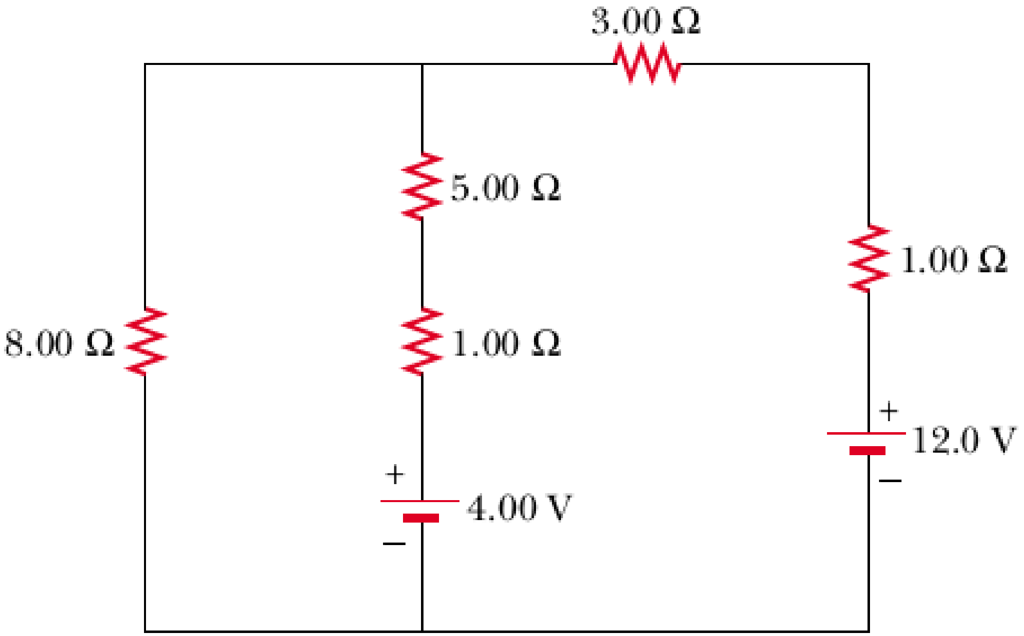
$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00 \times 10^{-4}\ \text{m}^2$$

(a) $$I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$$

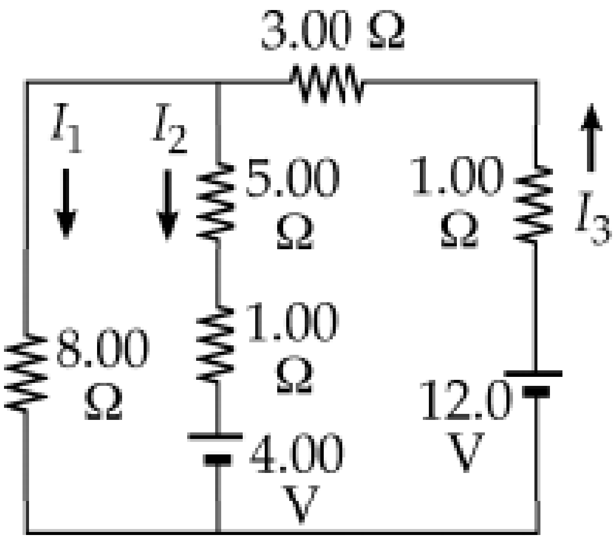
(b) $$J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00 \times 10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$$

Question 14

Determine the current in each branch of the circuit shown in Figure



Solution 14



We name currents I_1, I_2, and I_3 as shown.

From Kirchhoff's current rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0 \qquad (8.00)I_1 = 4.00 + (6.00)I_2.$$

Solution 14

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

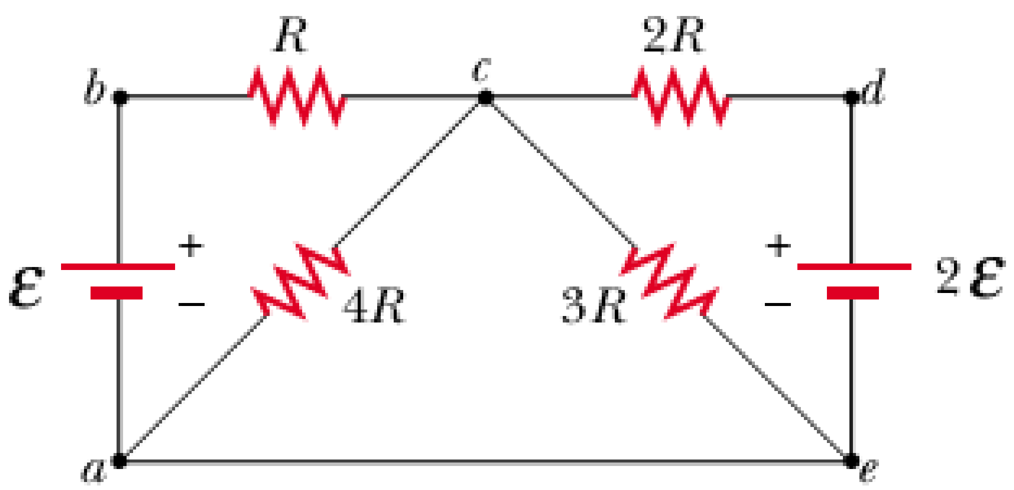
$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}. \qquad \text{Then} \qquad I_2 = 1.33(0.846\text{ A}) - 0.667$$

and $\quad I_3 = I_1 + I_2 \quad$ give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$.

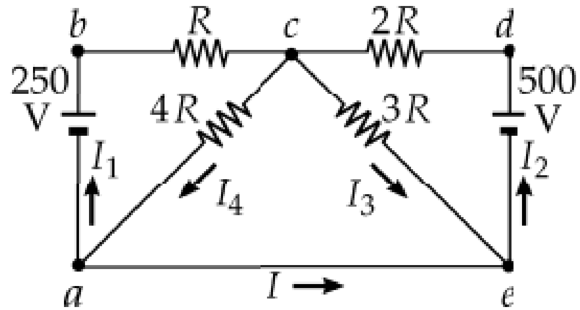
All currents are in the directions indicated by the arrows in the circuit diagram.

Question 15

Taking R=1.00 kΩ and ε=250 V in Figure, determine the direction and magnitude of the current in the horizontal wire between a and e.



Solution 15



(a)

Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

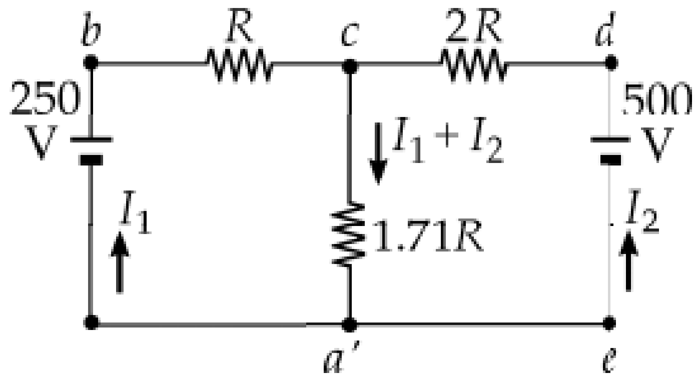
$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and

$$(1.71R)I_1 + (3.71R)I_2 = 500\,.$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

Solution 15

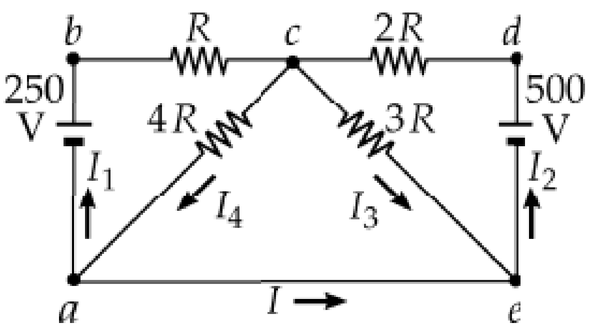


(b)

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\ \text{mA}$$

and $$I_2 = 130.0\ \text{mA}.$$



(a)

From Figure (b), $$V_c - V_a = (I_1 + I_2)(1.71R) = 240\ \text{V}.$$

Thus, from Figure (a), $$I_4 = \frac{V_c - V_a}{4R} = \frac{240\ \text{V}}{4\,000\ \Omega} = 60.0\ \text{mA}.$$

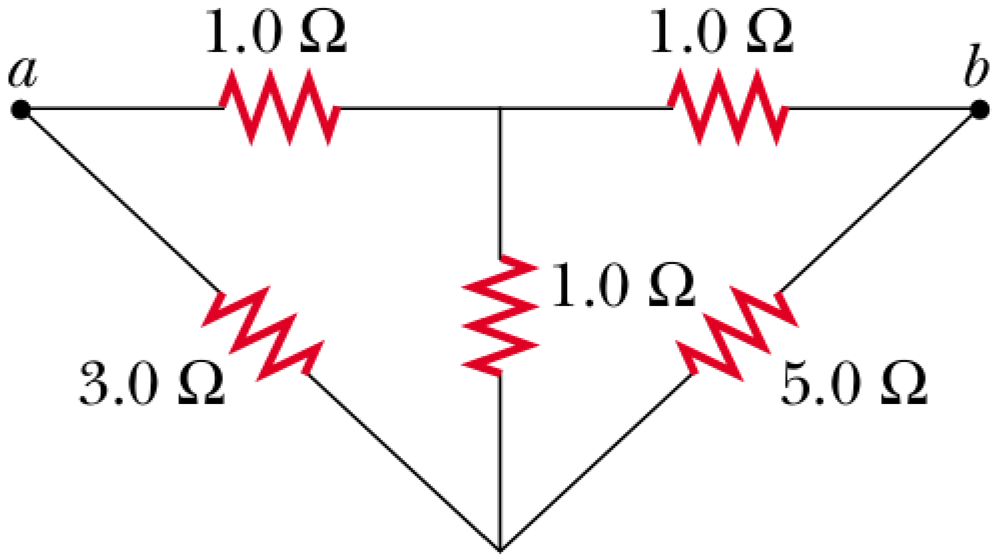
Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$$I = I_4 - I_1 = 60.0\ \text{mA} - 10.0\ \text{mA} = +50.0\ \text{mA},$$

or $$I = \boxed{50.0\ \text{mA from point } a \text{ to point } e}.$$

Question 16

For the network shown in Figure, show that the resistance Rab=(27/17)Ω.



Solution 16

$$\Delta V_{ab} = (1.00)I_1 + (1.00)(I_1 - I_2)$$

$$\Delta V_{ab} = (1.00)I_1 + (1.00)I_2 + (5.00)(I - I_1 + I_2)$$

$$\Delta V_{ab} = (3.00)(I - I_1) + (5.00)(I - I_1 + I_2)$$

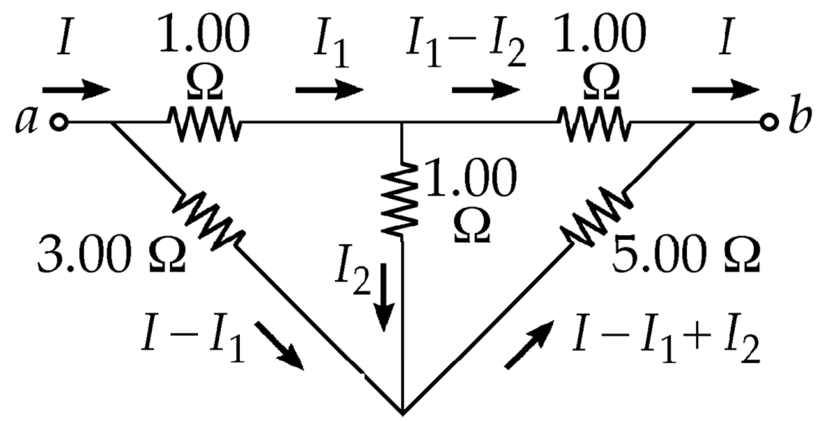
Let $I = 1.00\ \text{A}$, $I_1 = x$, and $I_2 = y$.

Then, the three equations become:

$\Delta V_{ab} = 2.00x - y$, or $y = 2.00x - \Delta V_{ab}$

$\Delta V_{ab} = -4.00x + 6.00y + 5.00$

and $\Delta V_{ab} = 8.00 - 8.00x + 5.00y$.



Solution 16

Substituting the first into the last two gives:

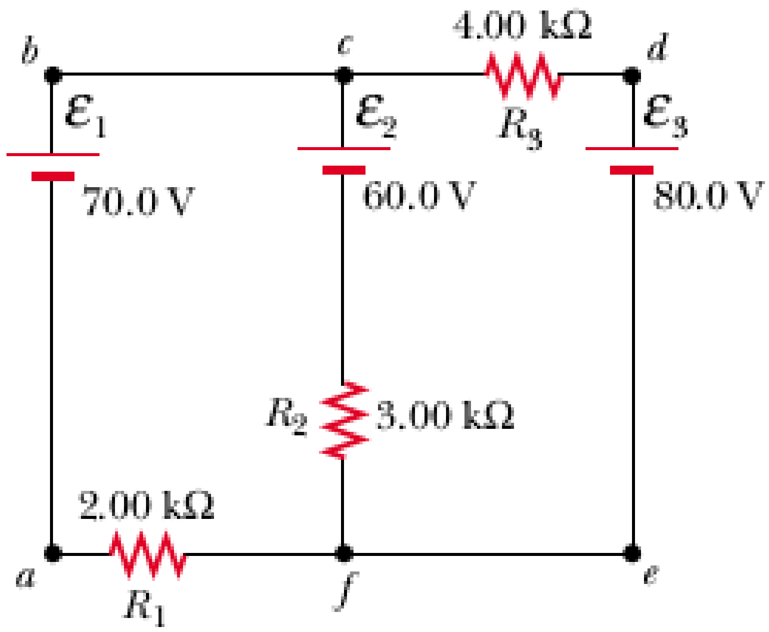
$7.00\Delta V_{ab} = 8.00x + 5.00$ and $6.00\Delta V_{ab} = 2.00x + 8.00$.

Solving these simultaneously yields $\Delta V_{ab} = \frac{27}{17}\ \mathrm{V}$.

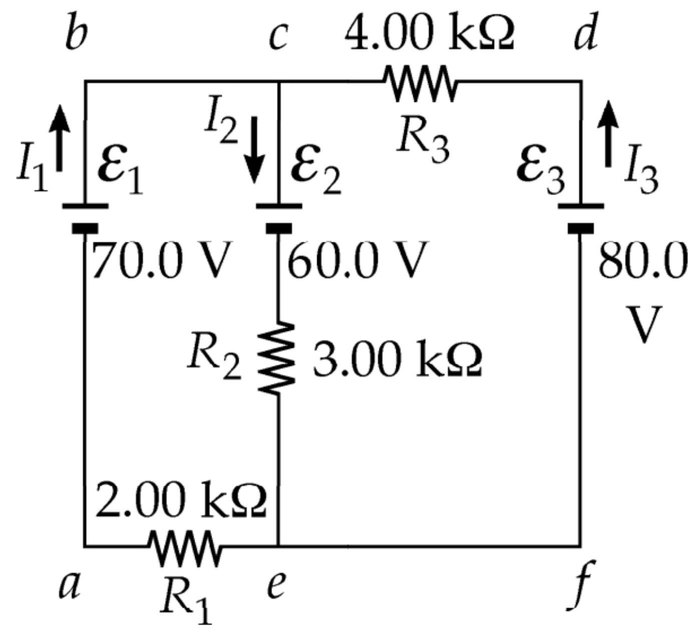
Then, $R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{\frac{27}{17}\ \mathrm{V}}{1.00\ \mathrm{A}}$ or $\boxed{R_{ab} = \frac{27}{17}\ \Omega}$.

Question 17

Using Kirchhoff's rules, (a) find the current in each resistor in Figure. (b) Find the potential difference between points c and f. Which point is at the higher potential?



Solution 17



We name the currents I_1, I_2, and I_3 as shown.

[1] $70.0 - 60.0 - I_2(3.00\text{ k}\Omega) - I_1(2.00\text{ k}\Omega) = 0$

[2] $80.0 - I_3(4.00\text{ k}\Omega) - 60.0 - I_2(3.00\text{ k}\Omega) = 0$

[3] $I_2 = I_1 + I_3$

Solution 17

(a) Substituting for I_2 and solving the resulting simultaneous equations yields

$$I_1 = \boxed{0.385 \text{ mA}} \ (\text{through } R_1)$$

$$I_3 = \boxed{2.69 \text{ mA}} \ (\text{through } R_3)$$

$$I_2 = \boxed{3.08 \text{ mA}} \ (\text{through } R_2)$$

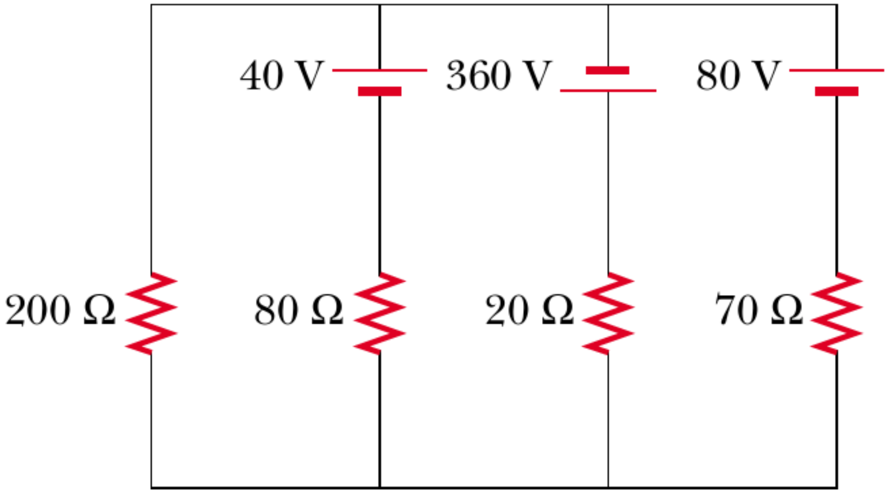
(b) $$\Delta V_{cf} = -60.0 \text{ V} - (3.08 \text{ mA})(3.00 \text{ k}\Omega) = \boxed{-69.2 \text{ V}}$$

$\boxed{\text{Point } c \text{ is at higher potential.}}$

3. WEEK

Question 1

In the circuit of Figure, determine the current in each resistor and the voltage across the 200-Ω resistor.



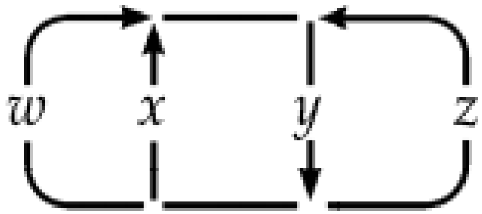
Solution 1

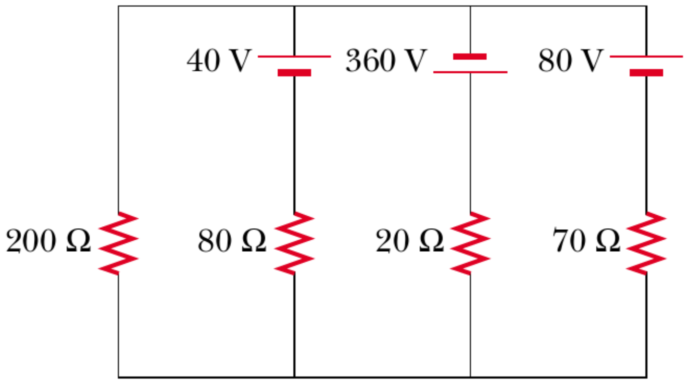
Name the currents as shown in the figure to the right. Then $w+x+z=y$. Loop equations are

$$-200w-40.0+80.0x=0$$

$$-80.0x+40.0+360-20.0y=0$$

$$+360-20.0y-70.0z+80.0=0$$





Solution 1

Eliminate y by substitution.

$$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x. $\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$

Eliminate $z = 17.5 - 13.5w$ to obtain

$$430 - 70.0w - 1\,575 + 1\,215w = 0$$

$$w = \frac{70.0}{70.0} = \boxed{1.00 \text{ A upward in } 200\ \Omega}$$

$$z = \boxed{4.00 \text{ A upward in } 70.0\ \Omega}$$

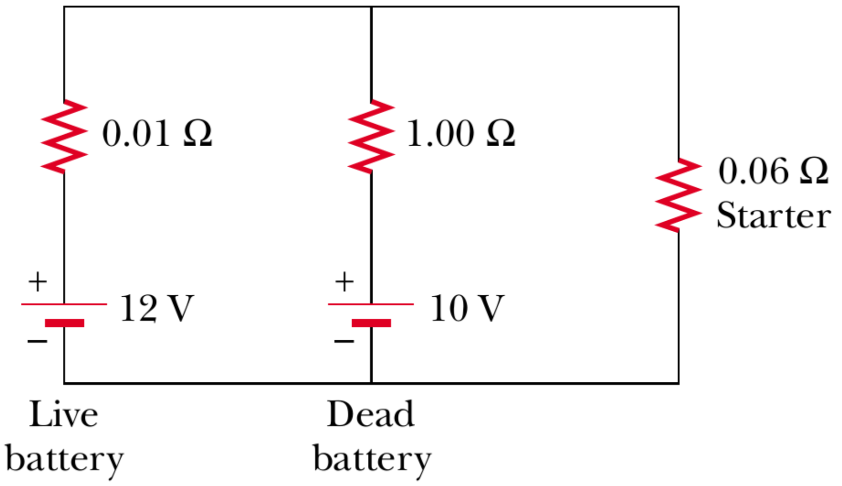
$$x = \boxed{3.00 \text{ A upward in } 80.0\ \Omega}$$

$$y = \boxed{8.00 \text{ A downward in } 20.0\ \Omega}$$

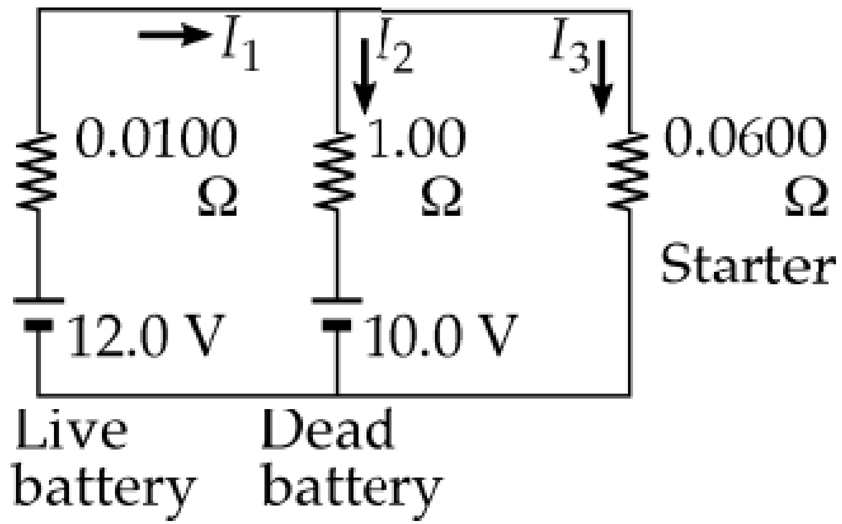
$$\Delta V = IR = (1.00 \text{ A})(200\ \Omega) = \boxed{200 \text{ V}}.$$

Question 2

A dead battery is charged by connecting it to the live battery of another car with jumper cables. Determine the current in the starter and in the dead battery.



Solution 2



Using Kirchhoff's rules,

$$12.0-(0.010\,0)I_1-(0.060\,0)I_3=0$$

$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

and $\quad I_1=I_2+I_3$

$$12.0-(0.010\,0)I_2-(0.070\,0)I_3=0$$

$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

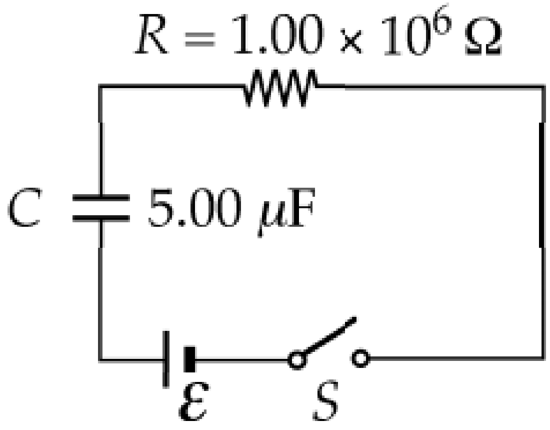
Solving simultaneously,

$I_2=\boxed{0.283\text{ A downward}}$ in the dead battery

and $\quad I_3=\boxed{171\text{ A downward}}$ in the starter.

Question 3

Consider a series RC circuit (see Fig.) for which R=1.00MΩ, C=5.00 μF, and ε=30.0V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) Find the current in the resistor 10.0s after the switch is closed.



Solution 3

(a) $$RC = \left(1.00 \times 10^{6}\ \Omega\right)\left(5.00 \times 10^{-6}\ \text{F}\right) = \boxed{5.00\ \text{s}}$$

(b) $$Q = C\varepsilon = \left(5.00 \times 10^{-6}\ \text{C}\right)(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$$

(c) $$I(t) = \frac{\varepsilon}{R} e^{-t/RC} = \left(\frac{30.0}{1.00 \times 10^{6}}\right)\exp\left[\frac{-10.0}{\left(1.00 \times 10^{6}\right)\left(5.00 \times 10^{-6}\right)}\right] = \boxed{4.06\ \mu\text{A}}$$

Question 4

A 2.00-nF capacitor with an initial charge of 5.10 μC is discharged through a 1.30-kΩ resistor.

(a) Calculate the current in the resistor 9.00 μs after the resistor is connected across the terminals of the capacitor.

(b) What charge remains on the capacitor after 8.00 μs?

(c) What is the maximum current in the resistor?

Solution 4

(a) $$I(t) = -I_0 e^{-t/RC}$$

$$I_0 = \frac{Q}{RC} = \frac{5.10 \times 10^{-6}\ \text{C}}{(1\,300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00 \times 10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00 \times 10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00 \times 10^{-9}\ \text{F}\right)}\right] = \boxed{0.235\ \mu\text{C}}$$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\ \text{A}}$.