Application of Physics II for

Midterm Exam
The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately $5.3 \times 10^{-11}$ m. Find the magnitudes of the electric force and the gravitational force between the two particles.
From Coulomb’s law, we find that the attractive electric force has the magnitude

\[ F_e = k_e \frac{|e|^2}{r^2} = \left( 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(5.3 \times 10^{-11} \text{ m})^2} = 8.2 \times 10^{-8} \text{ N} \]
we find that the gravitational force

\[ F_g = G \frac{m_em_p}{r^2} \]

\[ = \left( 6.7 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \]

\[ = \times \frac{(9.11 \times 10^{-31} \text{ kg}) (1.67 \times 10^{-27} \text{ kg})}{(5.3 \times 10^{-11} \text{ m})^2} \]

\[ = 3.6 \times 10^{-47} \text{ N} \]

The ratio \( F_e/F_g \approx 2 \times 10^{39} \). Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton’s law of gravitation and Coulomb’s law of electric forces. Other than magnitude, what is a fundamental difference between the two forces?
Question 2

Consider three point charges located at the corners of a right triangle as shown in Figure, where \( q_1 = q_3 = 5.0 \, \mu \text{C}, \quad q_2 = 2.0 \, \mu \text{C}, \) and \( a = 0.10 \, \text{m} \). Find the resultant force exerted on \( q_3 \).
\[ F_{23} = k_e \frac{|q_2| |q_3|}{a^2} \]
\[ = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{2.0 \times 10^{-6} \text{ C}}{0.10 \text{ m}} \right) \left( \frac{5.0 \times 10^{-6} \text{ C}}{(0.10 \text{ m})^2} \right) = 9.0 \text{ N} \]

\[ F_{13} = k_e \frac{|q_1| |q_3|}{(\sqrt{2}a)^2} \]
\[ = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{5.0 \times 10^{-6} \text{ C}}{2(0.10 \text{ m})^2} \right) = 11 \text{ N} \]

\[ F_{13x} = F_{13} \cos 45^\circ = 7.9 \text{ N} \]
\[ F_{13y} = F_{13} \sin 45^\circ = 7.9 \text{ N} \]

\[ F_{3x} = F_{13x} + F_{23x} = 7.9 \text{ N} + (-9.0 \text{ N}) = -1.1 \text{ N} \]
\[ F_{3y} = F_{13y} + F_{23y} = 7.9 \text{ N} + 0 = 7.9 \text{ N} \]

\[ \vec{F}_3 = (-1.1\hat{i} + 7.9\hat{j}) \text{ N} \]
Question 3

Three point charges lie along the $x$ axis as shown in Figure. The positive charge $q_1 = 15.0 \ \mu\text{C}$ is at $x = 2.00 \ \text{m}$, the positive charge $q_2 = 6.00 \ \mu\text{C}$ is at the origin, and the resultant force acting on $q_3$ is zero. What is the $x$ coordinate of $q_3$?
From Coulomb’s law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes

$$F_{13} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2} \quad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

For the resultant force on $q_3$ to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$, or

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$
Solution 3

\[(2.00 - x)^2|q_2| = x^2|q_1|\]
\[(4.00 - 4.00x + x^2)(6.00 \times 10^{-6} \text{ C}) = x^2(15.0 \times 10^{-6} \text{ C})\]

Solving this quadratic equation for \(x\), we find that \(x = 0.775 \text{ m.}\) Why is the negative root not acceptable?
Two identical small charged spheres, each having a mass of \(3.0 \times 10^{-2}\) kg, hang in equilibrium as shown in Figure 23. The length of each string is 0.15 m, and the angle \(\theta\) is 5.0°. Find the magnitude of the charge on each sphere.
Solution 4

\[ (1) \quad \sum F_x = T \sin \theta - F_e = 0 \]
\[ (2) \quad \sum F_y = T \cos \theta - mg = 0 \]

\[ F_e = mg \tan \theta \]
\[ = (3.0 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2) \tan 5.0^\circ \]
\[ = 2.6 \times 10^{-2} \text{ N} \]
Solution 4

From Coulomb’s law the magnitude of the electric force is

\[ F_e = k_e \frac{|q|^2}{r^2} \]

where \( r = 2a = 0.026 \text{ m} \) and \(|q|\) is the magnitude of the charge on each sphere. (Note that the term \(|q|^2\) arises here because the charge is the same on both spheres.) This equation can be solved for \(|q|^2\) to give

\[ |q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6 \times 10^{-2} \text{ N})(0.026 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \]

\[ |q| = 4.4 \times 10^{-8} \text{ C} \]
Question 5

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure. The rod has a total charge of $-7.50 \mu\text{C}$. Find the magnitude and direction of the electric field at $O$, the center of the semicircle.
Solution 5

Due to symmetry, \( E_y = \int dE_y = 0 \), and \( E_x = \int dE \sin \theta = k_e \int \frac{dq \sin \theta}{r^2} \)

where \( dq = \lambda ds = \lambda r d\theta \),

so that,

\[
E_x = \frac{k_e \lambda}{r} \int_0^\pi \sin \theta d\theta = \frac{k_e \lambda}{r} (-\cos \theta)|_0^\pi = \frac{2k_e \lambda}{r}
\]

where \( \lambda = \frac{q}{L} \) and \( r = \frac{L}{\pi} \).

Thus,

\[
E_x = \frac{2k_e q \pi}{L^2} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(7.50 \times 10^{-6} \text{ C})\pi}{(0.140 \text{ m})^2}
\]

Solving,

\[
E_x = 2.16 \times 10^7 \text{ N/C}.
\]

Since the rod has a negative charge, \( \mathbf{E} = (-2.16 \times 10^7 \hat{i}) \text{ N/C} = [21.6\hat{i} \text{ MN/C}] \).
Question 6

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure. The rod has a total charge of \(-7.50 \mu\text{C}\). Find the electric potential at O, the center of the semicircle.

Chapter 25
Solution 6

\[ V = \int dV = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r} \]

All bits of charge are at the same distance from \( O \).

So \[ V = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{R} \right) = \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( \frac{-7.50 \times 10^{-6} \text{ C}}{0.140 \text{ m/\pi}^2} \right) = -1.51 \text{ MV} \].
Question 7

Three equal positive charges $q$ are at the corners of an equilateral triangle of side $a$ as shown in Figure.

(a) Assume that the three charges together create an electric field. Sketch the field lines in the plane of the charges. Find the location of a point (other than 4) where the electric field is zero.

(b) What are the magnitude and direction of the electric field at $P$ due to the two charges at the base?
Solution 7

(a) The electric field has the general appearance shown. It is zero at the center, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where \( E = 0 \), but they are more difficult to find mathematically.
Solution 7

electric field at point $P$ can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$.

The electric field from a point charge is $\mathbf{E} = k_e \frac{q}{r^2} \mathbf{\hat{r}}$.

As shown in the solution figure at right,

$\mathbf{E}_1 = k_e \frac{q}{a^2}$ to the right and upward at $60^\circ$

$\mathbf{E}_2 = k_e \frac{q}{a}$ to the left and upward at $60^\circ$

$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2} \left[ (\cos 60^\circ \mathbf{\hat{i}} + \sin 60^\circ \mathbf{\hat{j}}) + (-\cos 60^\circ \mathbf{\hat{i}} + \sin 60^\circ \mathbf{\hat{j}}) \right] = k_e \frac{q}{a^2} \left[ 2\sin 60^\circ \mathbf{\hat{j}} \right]$

$= 1.73 k_e \frac{q}{a^2} \mathbf{\hat{j}}$
Question 8

A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in Figure. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?
Solution 8

From the free-body diagram shown,

\[ \sum F_y = 0 : \quad T \cos 15.0^\circ = 1.96 \times 10^{-2} \]

So

\[ T = 2.03 \times 10^{-2} \text{ N}. \]

From \( \sum F_x = 0 \), we have

\[ qE = T \sin 15.0^\circ \]

\[ q = \frac{T \sin 15.0^\circ}{E} = \frac{(2.03 \times 10^{-2} \text{ N}) \sin 15.0^\circ}{1.00 \times 10^3 \text{ N/C}} = 5.25 \times 10^{-6} \text{ C} = 5.25 \mu \text{C}. \]
Question 9

Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4 \text{ N/C}$ as shown in Figure.

Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.
Solution 9

(a) \[ A' = (10.0 \text{ cm})(30.0 \text{ cm}) \]
\[ A' = 300 \text{ cm}^2 = 0.0300 \text{ m}^2 \]
\[ \Phi_{E, A'} = EA' \cos \theta \]
\[ \Phi_{E, A'} = (7.80 \times 10^4)(0.0300) \cos 180^\circ \]
\[ \Phi_{E, A'} = -2.34 \text{ kN} \cdot \text{m}^2/\text{C} \]
(b) \[ \Phi_{E, A} = EA \cos \theta = \left(7.80 \times 10^4\right)(A) \cos 60.0^\circ \]

\[ A = (30.0 \text{ cm})(w) = (30.0 \text{ cm})\left(\frac{10.0 \text{ cm}}{\cos 60.0^\circ}\right) = 600 \text{ cm}^2 = 0.0600 \text{ m}^2 \]

\[ \Phi_{E, A} = \left(7.80 \times 10^4\right)(0.0600) \cos 60.0^\circ = +2.34 \text{ kN} \cdot \text{m}^2/\text{C} \]

(c) The bottom and the two triangular sides all lie parallel to \( \mathbf{E} \), so \( \Phi_E = 0 \)

\[ \Phi_{E, \text{total}} = -2.34 \text{ kN} \cdot \text{m}^2/\text{C} + 2.34 \text{ kN} \cdot \text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}. \]
Question 10

The following charges are located inside a submarine: 5.00 µC, -9.00 µC, 27.0 µC, and -84.0 µC.

(a) Calculate the net electric flux through the hull of the submarine.

(b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?
Solution 10

(a) \[ \Phi_E = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{(+5.00 \, \mu C - 9.00 \, \mu C + 27.0 \, \mu C - 84.0 \, \mu C)}{8.85 \times 10^{-12} \, \text{C}^2/\text{N} \cdot \text{m}^2} = -6.89 \times 10^6 \, \text{N} \cdot \text{m}^2/\text{C}^2 \]

\[ \Phi_E = \boxed{-6.89 \, \text{MN} \cdot \text{m}^2/\text{C}} \]

(b) Since the net electric flux is negative, more lines enter than leave the surface.
Question 11

Four closed surfaces, S1 through S4, together with the charges -2Q, Q, and -Q are sketched in Figure. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.
Solution 11

\[ \Phi_E = \frac{Q_{\text{in}}}{\varepsilon_0} \]

Through \( S_1 \)
\[ \Phi_E = \frac{-2Q + Q}{\varepsilon_0} \]
\[ \Phi_E = \frac{-Q}{\varepsilon_0} \]

Through \( S_2 \)
\[ \Phi_E = \frac{+Q - Q}{\varepsilon_0} \]
\[ \Phi_E = \frac{0}{\varepsilon_0} \]

Through \( S_3 \)
\[ \Phi_E = \frac{-2Q + Q - Q}{\varepsilon_0} \]
\[ \Phi_E = \frac{-2Q}{\varepsilon_0} \]

Through \( S_4 \)
\[ \Phi_E = \frac{0}{\varepsilon_0} \]
Question 12

An uncharged nonconducting hollow sphere of radius 10.0 cm surrounds a 10.0-\(\mu\)C charge located at the origin of a cartesian coordinate system. A drill with a radius of 1.00 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.
Solution 12

\[ \Phi_{E, \text{hole}} = \mathbf{E} \cdot \mathbf{A}_{\text{hole}} = \left( \frac{k_e Q}{R^2} \right) \left( \pi r^2 \right) \]

\[ = \left( \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \times 10^{-6} \text{ C})}{(0.100 \text{ m})^2} \right) \pi \left( 1.00 \times 10^{-3} \text{ m} \right)^2 \]

\[ \Phi_{E, \text{hole}} = 28.2 \text{ N} \cdot \text{m}^2/\text{C} \]
Question 13

A charge of 170μC is at the center of a cube of edge 80.0 cm.

(a) Find the total flux through each face of the cube.

(b) Find the flux through the whole surface of the cube.

(c) What If? Would your answers to parts (a) or (b) change if the charge were not at the center? Explain
Solution 13

\[ \Phi_E = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{170 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 1.92 \times 10^7 \text{ N} \cdot \text{m}^2/\text{C} \]

(a) \( (\Phi_E)_{\text{one face}} = \frac{1}{6} \Phi_E = \frac{1.92 \times 10^7 \text{ N} \cdot \text{m}^2/\text{C}}{6} \)

\( (\Phi_E)_{\text{one face}} = 3.20 \text{ MN} \cdot \text{m}^2/\text{C} \)

(b) \( \Phi_E = 19.2 \text{ MN} \cdot \text{m}^2/\text{C} \)
Solution 13

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.
Question 14

A solid insulating sphere of radius \( a \) carries a net positive charge \( 3Q \), uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius \( b \) and outer radius \( c \), and having a net charge \( -Q \), as shown in Figure.
(a) Construct a spherical gaussian surface of radius $r-c$ and find the net charge enclosed by this surface.

(b) What is the direction of the electric field at $r>c$?

(c) Find the electric field at $r>c$.

(d) Find the electric field in the region with radius $r$ where $c>r>b$.

(e) Construct a spherical gaussian surface of radius $r$, where $c>r>b$, and find the net charge enclosed by this surface.

(f) Construct a spherical gaussian surface of radius $r$, where $b>r>a$, and find the net charge enclosed by this surface.
(g) Find the electric field in the region \( b > r > a \).

(h) Construct a spherical gaussian surface of radius \( r < a \), and find an expression for the net charge enclosed by this surface, as a function of \( r \). Note that the charge inside this surface is less than \( 3Q \).

(i) Find the electric field in the region \( r < a \).

(j) Determine the charge on the inner surface of the conducting shell.

(k) Determine the charge on the outer surface of the conducting shell.

(l) Make a plot of the magnitude of the electric field versus \( r \).
Solution 14

(a) \[ q_{in} = +3Q - Q = +2Q \]

(b) The charge distribution is spherically symmetric and \( q_{in} > 0 \). Thus, the field is directed radially outward.

(c) \[ E = \frac{k e q_{in}}{r^2} = \frac{2k e Q}{r^2} \ 	ext{for} \ r \geq c. \]
Solution 14

(d) Since all points within this region are located inside conducting material, \( E = 0 \) for \( b < r < c \).

(e) \( \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = 0 \Rightarrow q_{\text{in}} = \varepsilon_0 \Phi_E = 0 \)

(f) \( q_{\text{in}} = +3Q \)

(g) \( E = \frac{k_e q_{\text{in}}}{r^2} = \frac{3k_e Q}{r^2} \) (radially outward) for \( a \leq r < b \).

(h) \( q_{\text{in}} = \rho V = \left( \frac{+3Q}{\frac{4}{3} \pi a^3} \right) \left( \frac{4}{3} \pi r^3 \right) = +3Q \frac{r^3}{a^3} \)

(i) \( E = \frac{k_e q_{\text{in}}}{r^2} = \frac{k_e}{r^2} \left( +3Q \frac{r^3}{a^3} \right) = 3k_e Q \frac{r}{a^3} \) (radially outward) for \( 0 \leq r \leq a \).
Solution 14

(j) From part (d), $E = 0$ for $b < r < c$. Thus, for a spherical gaussian surface with $b < r < c$, $q_{\text{in}} = +3Q + q_{\text{inner}} = 0$ where $q_{\text{inner}}$ is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = -3Q$.

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -Q$, we have $q_{\text{outer}} = -Q - q_{\text{inner}} = -Q - (-3Q) = +2Q$.

(l) This is shown in the figure to the right.
Question 15

An infinitely long line charge having a uniform charge per unit length $\lambda$ lies a distance $d$ from point O as shown in Figure. Determine the total electric flux through the surface of a sphere of radius $R$ centered at O resulting from this line charge. Consider both cases, where $R<d$ and $R>d$. 

Chapter 24
Solution 15

If $R \leq d$, the sphere encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = 0$.

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$.

So $\Phi_E = \frac{2\lambda \sqrt{R^2 - d^2}}{\epsilon_0}$. 
Question 16

An electron moving parallel to the x-axis has an initial speed of $3.70 \times 10^6 \text{ m/s}$ at the origin. Its speed is reduced to $1.40 \times 10^5 \text{ m/s}$ at the point $x=2.00 \text{ cm}$. Calculate the potential difference between the origin and that point. Which point is at the higher potential?
Solution 16

\[ \Delta U = -\frac{1}{2}m(v_f^2 - v_i^2) \]

\[ = -\frac{1}{2}(9.11 \times 10^{-31} \text{ kg}) \left[ (1.40 \times 10^5 \text{ m/s})^2 - (3.70 \times 10^6 \text{ m/s})^2 \right] \]

\[ = 6.23 \times 10^{-18} \text{ J} \]

\[ \Delta U = q\Delta V : \quad +6.23 \times 10^{-18} = (-1.60 \times 10^{-19})\Delta V \]

\[ \Delta V = -38.9 \text{ V. The origin is at highest potential.} \]
Question 17

Given two 2.00-μC charges, as shown in Figure and a positive test charge q = 1.28 × 10^{-18} C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?
Solution 17

(a) Since the charges are equal and placed symmetrically, \( F = 0 \).

(b) Since \( F = qE = 0 \), \( E = 0 \).

(c) \[
V = 2k_e \frac{q}{r} = 2 \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{2.00 \times 10^{-6} \text{ C}}{0.800 \text{ m}}\right)
\]

\[
V = 4.50 \times 10^4 \text{ V} = 45.0 \text{ kV}
\]
Question 18

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120V.

(b) Calculate the speed of an electron that is accelerated through the same potential difference.
Solution 18

(a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

\[ K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \]
\[ 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0 \]
\[ \left(1.60 \times 10^{-19} \ \text{C}\right)(120 \ \text{V})\left(\frac{1 \text{J}}{1 \text{V} \cdot \text{C}}\right) = \frac{1}{2} \left(1.67 \times 10^{-27} \ \text{kg}\right)v_p^2 \]

\[ v_p = 1.52 \times 10^5 \ \text{m/s} \]
(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V:

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$0 + 0 + 0 = \frac{1}{2} m v_e^2 + qV$$

$$0 = \frac{1}{2} \left(9.11 \times 10^{-31} \text{ kg}\right) v_e^2 + \left(-1.60 \times 10^{-19} \text{ C}\right)(120 \text{ J/C})$$

$$v_e = 6.49 \times 10^6 \text{ m/s}$$
Question 19

A wire having a uniform linear charge density $\lambda$ is bent into the shape shown in Figure. Find the electric potential at point O.
Solution 19

\[ V = k_e \int \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x} \]

\[ V = -k_e \lambda \ln(-x) \bigg|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x \bigg|_{R}^{3R} \]

\[ V = k_e \ln \frac{3R}{R} + k_e \lambda \pi + k_e \ln 3 = k_e \lambda (\pi + 2 \ln 3) \]
Question 20

A rod of length L (Fig.) lies along the x-axis with its left end at the origin. It has a nonuniform charge density \( \lambda = \alpha x \), where \( \alpha \) is a positive constant. (a) What are the units of \( \alpha \)? (b) Calculate the electric potential at A.
Solution 20

(a) \[ [\alpha] = \left[ \frac{\lambda}{x} \right] = \frac{C}{\text{m}} \cdot \left( \frac{1}{\text{m}} \right) = \frac{C}{\text{m}^2} \]

(b) \[ V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda dx}{r} = k_e \alpha \int_{0}^{L} \frac{xdx}{d+x} = k_e \alpha \left[ L - d \ln \left( 1 + \frac{L}{d} \right) \right] \]
A rod of length $\ell$ has a uniform positive charge per unit length $\lambda$ and a total charge $Q$. Calculate the electric field at a point $P$ that is located along the long axis of the rod and a distance $a$ from one end.
per unit length $\lambda$, the charge $dq$ on the small segment is $dq = \lambda \, dx$.

The field $dE$ due to this segment at $P$ is in the negative $x$ direction (because the source of the field carries a positive charge $Q$), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e \lambda \frac{dx}{x^2}$$
Solution 21

\[ E = \int_{a}^{\ell+a} k_e \lambda \frac{dx}{x^2} \]

where the limits on the integral extend from one end of the rod \((x = a)\) to the other \((x = \ell + a)\). The constants \(k_e\) and \(\lambda\) can be removed from the integral to yield

\[ E = k_e \lambda \int_{a}^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x}\right]_a^{\ell+a} \]

\[ = k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \frac{k_e Q}{a(\ell + a)} \]
After this slide there are summaries of three chapter for about related midterm exam.
Chapter 23

**SUMMARY**

**Electric charges** have the following important properties:

- Charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

**Conductors** are materials in which electrons move freely. **Insulators** are materials in which electrons do not move freely.

**Coulomb's law** states that the electric force exerted by a charge $q_1$ on a second charge $q_2$ is

$$
\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}
$$

where $r$ is the distance between the two charges and $\hat{r}$ is a unit vector directed from $q_1$ toward $q_2$. The constant $k_e$, which is called the Coulomb constant, has the value $k_e = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. 
The smallest unit of free charge \( e \) known to exist in nature is the charge on an electron \((-e)\) or proton \((+e)\), where \( e = 1.60219 \times 10^{-19} \) C.

The electric field \( \mathbf{E} \) at some point in space is defined as the electric force \( \mathbf{F}_e \) that acts on a small positive test charge placed at that point divided by the magnitude \( q_0 \) of the test charge:

\[
\mathbf{E} = \frac{\mathbf{F}_e}{q_0}
\]

Thus, the electric force on a charge \( q \) placed in an electric field \( \mathbf{E} \) is given by

\[
\mathbf{F}_e = q\mathbf{E}
\]
At a distance $r$ from a point charge $q$, the electric field due to the charge is given by

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector directed from the charge toward the point in question. The electric field is directed radially outward from a positive charge and radially inward toward a negative charge.

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

The electric field at some point due to a continuous charge distribution is

$$\mathbf{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

where $dq$ is the charge on one element of the charge distribution and $r$ is the distance from the element to the point in question.

**Electric field lines** describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of $\mathbf{E}$ in that region.

A charged particle of mass $m$ and charge $q$ moving in an electric field $\mathbf{E}$ has an acceleration

$$\mathbf{a} = \frac{q \mathbf{E}}{m}$$
**SUMMARY**

**Electric flux** is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle $\theta$ with the normal to a surface of area $A$, the electric flux through the surface is

$$\Phi_E = EA \cos \theta$$

In general, the electric flux through a surface is

$$\Phi_E = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$
## Typical Electric Field Calculations Using Gauss’s Law

<table>
<thead>
<tr>
<th>Charge Distribution</th>
<th>Electric Field</th>
<th>Location</th>
</tr>
</thead>
</table>
| Insulating sphere of radius $R$, uniform charge density, and total charge $Q$ | \[
\begin{aligned}
k_e \frac{Q}{r^2} & \quad r > R \\
k_e \frac{Q}{R^2} r & \quad r < R
\end{aligned}
\] |          |
| Thin spherical shell of radius $R$ and total charge $Q$ | \[
\begin{aligned}
k_e \frac{Q}{r^2} & \quad r > R \\
0 & \quad r < R
\end{aligned}
\] |          |
| Line charge of infinite length and charge per unit length $\lambda$ | \[2k_e \frac{\lambda}{r}\] | Outside the line |
| Infinite charged plane having surface charge density $\sigma$ | \[\frac{\sigma}{2\epsilon_0}\] | Everywhere outside the plane |
| Conductor having surface charge density $\sigma$ | \[
\begin{aligned}
\frac{\sigma}{\epsilon_0} & \quad \text{Just outside the conductor} \\
0 & \quad \text{Inside the conductor}
\end{aligned}
\] |          |
You should be able to apply Equations 24.2 and 24.3 in a variety of situations, particularly those in which symmetry simplifies the calculation.

Gauss’s law says that the net electric flux $\Phi_E$ through any closed gaussian surface is equal to the net charge $q_{in}$ inside the surface divided by $\epsilon_0$:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

Using Gauss’s law, you can calculate the electric field due to various symmetric charge distributions. Table 24.1 lists some typical results.

A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on the conductor resides entirely on its surface.
3. The electric field just outside the conductor is perpendicular to its surface and has a magnitude $\sigma/\epsilon_0$, where $\sigma$ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.
Chapter 25

**SUMMARY**

When a positive test charge $q_0$ is moved between points $A$ and $B$ in an electric field $\mathbf{E}$, the change in the potential energy of the charge-field system is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The electric potential $V = U/q_0$ is a scalar quantity and has the units of J/C, where 1 J/C $= 1$ V.

The potential difference $\Delta V$ between points $A$ and $B$ in an electric field $\mathbf{E}$ is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The potential difference between two points $A$ and $B$ in a uniform electric field $\mathbf{E}$, where $\mathbf{s}$ is a vector that points from $A$ to $B$ and is parallel to $\mathbf{E}$ is

$$\Delta V = -Ed$$

where $d = |\mathbf{s}|$. 
An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance $r$ from the charge is

$$V = k_e \frac{q}{r}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance $r_{12}$ is

$$U = k_e \frac{q_1 q_2}{r_{12}}$$
This energy represents the work done by an external agent when the charges are brought from an infinite separation to the separation \( r_{12} \). We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

If we know the electric potential as a function of coordinates \( x, y, z \), we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the \( x \) component of the electric field is

\[
E_x = -\frac{dV}{dx}
\]

The **electric potential due to a continuous charge distribution** is

\[
V = k_e \int \frac{dq}{r}
\]

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.
# Electric Potential Due to Various Charge Distributions

<table>
<thead>
<tr>
<th>Charge Distribution</th>
<th>Electric Potential</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniformly charged ring of radius $a$</td>
<td>$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$</td>
<td>Along perpendicular central axis of ring, distance $x$ from ring center</td>
</tr>
<tr>
<td>Uniformly charged disk of radius $a$</td>
<td>$V = 2\pi k_e \sigma [(x^2 + a^2)^{1/2} - x]$</td>
<td>Along perpendicular central axis of disk, distance $x$ from disk center</td>
</tr>
<tr>
<td>Uniformly charged, insulating solid sphere of radius $R$ and total charge $Q$</td>
<td>$\begin{cases} V = k_e \frac{Q}{r} &amp; r \geq R \ V = \frac{k_e Q}{2R} \left( 3 - \frac{r^2}{R^2} \right) &amp; r &lt; R \end{cases}$</td>
<td></td>
</tr>
<tr>
<td>Isolated conducting sphere of radius $R$ and total charge $Q$</td>
<td>$\begin{cases} V = k_e \frac{Q}{r} &amp; r &gt; R \ V = k_e \frac{Q}{R} &amp; r \leq R \end{cases}$</td>
<td></td>
</tr>
</tbody>
</table>